43. If a spring extends by x on loading, then the energy stored by the spring is [if T is tension in the spring and K is spring constant]

(a) $T^2/2x$ (b) $T^2/2K$ (c) $2K/T^2$ (d) $2T^2/K$.

Solution:- $U = \frac{1}{2} K x^2$ | $\because T = Kx$, $x = T/K$

$$U = \frac{1}{2} \cancel{K} \cdot \frac{T^2}{K^{\cancel{2}}}$$

$$U = T^2/2K.$$

44. Two springs of constant K_1 and K_2 have equal highest velocities, when executing S.H.M. Then, the ratio of their amplitudes (given their masses are equal) will be

(a) K_1/K_2 (b) $\left(\frac{K_1}{K_2}\right)^{1/2}$ (c) $\frac{K_2}{K_1}$ (d) $\left(\frac{K_2}{K_1}\right)^{1/2}$.

Solution:- Kinetic energies of both masses are equal.

Therefore

$$\frac{1}{\cancel{2}} K_1 a_1^2 = \frac{1}{\cancel{2}} K_2 a_2^2$$

$$\text{or } \frac{a_1}{a_2} = \left(\frac{K_2}{K_1}\right)^{1/2}$$

45 Two springs of spring constants 1500 N/m and 3000 N/m respectively are stretched with the same force. If they will have potential energy in the ratio:

(a) 4:1 (b) 1:4 (c) 2:1 (d) 1:2

Solution:-

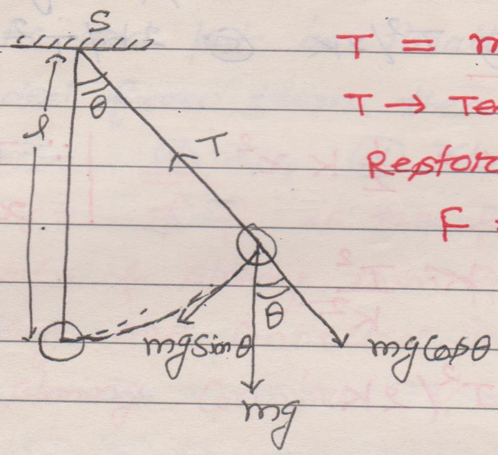
$$\frac{U_1}{U_2} = \frac{\frac{1}{\cancel{2}} K_1 x_1^2}{\frac{1}{\cancel{2}} K_2 x_2^2} = \frac{K_1}{K_2} \cdot \left(\frac{x_1}{x_2}\right)^2 \quad \text{—} (1)$$

$$f = K_1 x_1 = K_2 x_2 \quad \text{or} \quad \frac{x_1}{x_2} = \frac{K_2}{K_1} \quad \text{—} (2)$$

from (1) & (2)

$$\frac{U_1}{U_2} = \frac{K_1}{K_2} \times \frac{K_2^2}{K_1^2} = \frac{K_2}{K_1} = \frac{3000}{1500} = 2:1$$

Simple Pendulum :



$T = mg\cos\theta$

$T \rightarrow$ Tension

Restoring force

$F = -mg\sin\theta.$

Time Period:

$$\boxed{T = 2\pi\sqrt{l/g}}$$

* ~~The~~ The time period of a simple pendulum is independent of amplitude, if the amplitude is sma while if the amplitude is large then the time per is given by

$$\# \quad \boxed{T = 2\pi\sqrt{\frac{l}{g}\left[1+\frac{1}{2^2}\sin^2\frac{\theta_m}{2}+\frac{1}{2^2}\cdot\frac{3^2}{4^2}\sin^4\frac{\theta_m}{2}+\cdots\right.}}$$

* The time period of the simple Pendulum is independen of the mass and material of the bob.

* If l is constant then $T \propto \frac{1}{\sqrt{g}}$

* If g is constant then $T \propto \sqrt{l}$.

* If l is comparable with the radius of the earth, then the time period of simple pendulum is given

$$\boxed{T = 2\pi\sqrt{\frac{R}{\left(1+\frac{R}{l}\right)g}}}$$

* If $l = R$ then

$$\boxed{T = 2\pi\sqrt{\frac{R}{2g}} = 59.8 \text{ minute}}$$

* If $l = \infty$ then

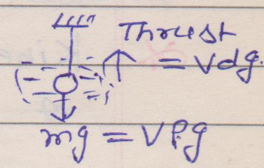
$$\boxed{T = 2\pi\sqrt{R/g} = 84.6 \text{ minute.}}$$

* Time period of simple Pendulum when its angular amplitude (θ_0) is large is given by:

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}\left(1 + \theta_0^2/16\right)}}$$

* If a simple Pendulum whose bob of density ρ is made to oscillate in a liquid of density d then its time Period

$$\boxed{T = 2\pi\sqrt{\frac{l}{(1 - d/\rho)g}}}$$



* If the liquid is highly viscous, then due to high resistance of the medium, the amplitude of simple Pendulum decreases exponentially and $T = \infty$. It means the pendulum does not oscillate.
* If bob of simple pendulum is negatively charged and is made to oscillate above the negatively charged Plate, then the effective acc of the bob decreases and time Period increases:-

$$\boxed{T = 2\pi\sqrt{\frac{l}{\left(g - \frac{qE}{m}\right)}}}$$

* If the bob of simple Pendulum is negatively Charged and a positively Charged Plate is Placed below it, then the effective acceleration of bob increases and time Period decreases:

$$\boxed{T = 2\pi\sqrt{\frac{l}{(g + 2E/m)}}}$$

* work done in Producing an angular displacement θ to the bob of simple Pendulum of mass from its mean Position is:

$$\boxed{W = mgh = mgl(1-\cos\theta)}$$

where l is the length of simple pendulum. This work done appears as potential energy of the pendulum at height 'h' from mean position.

* Kinetic energy of Pendulum at angular displacement θ is given by.

$$\boxed{E_K = mgl\cos\theta}$$

* If a lift is accelerated upwards with acceleration 'a' then the time Period of simple pendulum in the lift is

$$\boxed{T = 2\pi\sqrt{\frac{l}{g+a}}}$$

* If a lift is accelerated downwards with acceleration a then time period of a simple Pendulum in the lift is

$$\boxed{T = 2\pi\sqrt{\frac{l}{g-a}}}$$

* If a lift is moving upwards or downwards with a constant velocity v, then $a = 0$

$$\boxed{\therefore \; T = 2\pi\sqrt{\frac{l}{g}}}$$

* When a lift is freely falling with acceleration g; then

$$\boxed{T = 2\pi\sqrt{\frac{l}{g-g}} = \infty}$$

* When a vehicle (Bus, Car, train) is moving with an acceleration 'a' in the horizontal direction then the time Period of simple Pendulum on vehicle is:

$$\boxed{T' = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}}$$

* A piece of wood has dimensions $a \times b \times c$ and density ρ. It is floating in a liquid of density d such that side 'a' is vertical. It is now pushed down gently and released. The time Period is

$$\boxed{T = 2\pi\sqrt{\rho a/gd}}$$

Let the Piece of wood be floating with side (a) vertical and its distance x be pushed in liquid. Then, buoyancy force

$= b \cdot c \cdot x d \cdot g$ [d - density of liquid for water $d = 1$

The mass of piece of wood $= abc\rho$.

So acceleration $= \frac{-bcxdg}{abc \cdot \rho} = \frac{xdg}{a\rho} = \frac{xg}{a\rho}$

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{x}{xgd/a\rho}}$$

$$T = 2\pi\sqrt{\frac{a\rho}{gd}}$$

* A closed container of mass 'm' Kg and area of cross-section A floats uprights in a liquid of density ρ Kg/m^3. from its equilibrium position it is pushed down a bit and then released. It will oscillate up and down with a period!

$$\boxed{T = 2\pi\sqrt{\frac{m}{A\rho g}}}$$

Let the body be depressed upto a depth 'y' meter in water. Then
weight of water displaced = upward thrust

$$F = -Ay\rho g \quad \text{—①}$$
$$\& \quad F = -Ky \quad \text{—②}$$

from ① & ② $K = A\rho g$.

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{A\rho g}}}$$

* One end of long metallic wire of length L is tied to the ceiling. The other end is tied to massless spring of spring constant 'K'. A mass 'm' hangs freely from the free end of the spring. A area of cross-section and Young's Modulus of the wire are 'A' and 'Y' respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

$$\boxed{T = 2\pi\sqrt{\frac{(YA+KL)m}{YAK}}}$$

Solution:- $Y = \frac{FL}{A \cdot \Delta L}$ or $\frac{F}{\Delta L} = \frac{YA}{L}$.

force constant of wire $K_1 = \frac{F}{\Delta L} = \frac{YA}{L}$.

and spring of force constant K are in series the

Combined force constant $K_2 = \frac{K_1 K}{K_1 + K}$

$$K_2 = \frac{YA/L \cdot K}{\frac{YA}{L} + K} = \frac{YAK}{YA + KL}.$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K_2}} = \sqrt{\frac{m(YA+KL)}{YAK.}}}$$

* A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance X. Now the combined mass will oscillate on the spring with period

$$\boxed{T = 2\pi\sqrt{\frac{(M+m)x}{mg}}.}$$

Solution:— spring constant $K = \frac{mg}{x}$.

$$\boxed{T = 2\pi\sqrt{\frac{(M+m)}{K}} = 2\pi\sqrt{\frac{(M+m)x}{mg.}}}$$

* A uniform cylinder of length L and mass M having cross sectional area 'A' is suspended with its vertical length, from a fixed point by a massless spring, such that it is half submerged in a liquid of density 'd' at equilibrium position. When the cylinder is given a small downward push and released, it starts oscillating vertically with a small amplitude. If the force constant of the spring 'K' the Time Period of oscillation of the cylinder is:

$$\boxed{T = 2\pi\sqrt{\frac{M}{(K+Adg)}}}$$

Solution:— When the cylinder is given a small downward displacement, say y, the additional restoring force is due to (i) additional extension y, which is $F_1 = Ky$.
(ii) additional buoyancy, which is $f_2 = Aydg$.

$\therefore$ Total resoring force $f = f_1 + f_2$

$$f = (K + Adg)y \quad \text{--- (1)}$$

& $$f' = -K'y \quad \text{--- (2)}$$

from (1) & (2) $K' = K + Adg$.

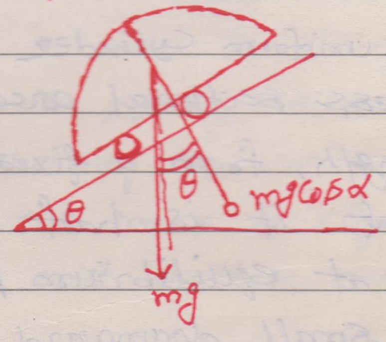
$$\boxed{\therefore T = 2\pi\sqrt{\frac{M}{K'}} = 2\pi\sqrt{\frac{M}{K + Adg}}}$$

* The Period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination θ is given by

$$\boxed{T = 2\pi\sqrt{\frac{L}{g\cos\theta}}}$$

Solution:—

$$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

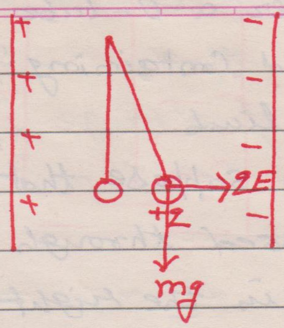


* A simple pendulum has a length l cm. Mass of bob ~~is~~ m grams. The bob is given a charge of $+q$ stat Coulombs. The pendulum is suspended between the plates of a charged parallel plate capacitor. If E is the electric Intensity between the plates as shown in fig. then the time period is given by

$$\boxed{T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}}$$

$$ma = f = \sqrt{m^2g^2 + (qE)^2}$$

$$a = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$



$$\therefore \boxed{T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}}$$

* A test tube with some lead shots in it has total mass m. Its cross section is a. It floats upright in a liquid of density d. From its equilibrium position, it is pushed down a bit and then released. It will oscillate up and down with a Period:

$$\boxed{T = 2\pi\sqrt{\frac{m}{a\,g.d}}}$$

Solution:- When tube is pushed downwards through distance x, Buoyancy force $f = axdg$. ——①

& $f = Kx$ ——②

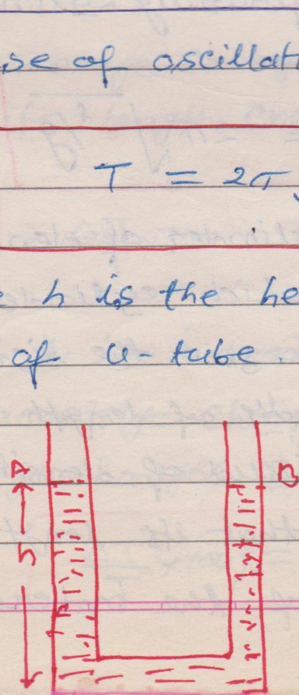
from ① & ② $K = adg$.

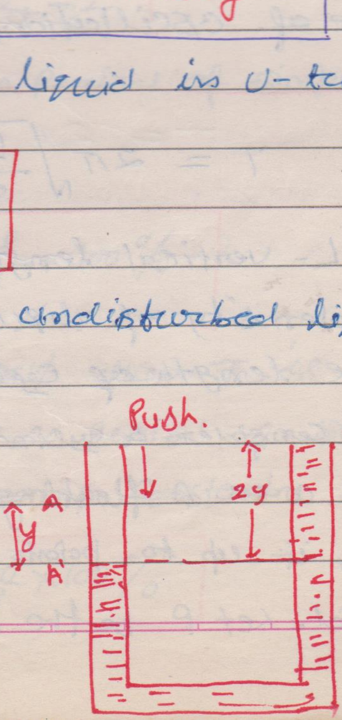
$$\boxed{\therefore \; T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{m}{adg}}}$$

* In case of oscillations of liquid in U-tube, time Period

$$\boxed{T = 2\pi\sqrt{h/g}}$$

where h is the height of undisturbed liquid in each limb of U-tube.





* The graph between l & T^2 in case of simple Pendulum is straight line
* graph between l & T is a Parabola.
* $l-T$ & $l-T^2$ graph intersect at $T = 1$ sec.

Consider a U-tube of uniform area of cross-section A and containing liquid of density ρ up to height h in each limb.

Suppose that the liquid in the left limb is depressed through distance y. As a result the liquid will rise in the right limb by an equal amount 'y'. The weight of the liquid column '$2y$' will provide the restoring force to the liquid so as to make the leaves in the two limbs again equal. Now the restoring force

$$f = -\text{weight of liquid of height } 2y$$
$$= -\text{Volume} \times \text{density} \times g.$$
$$= -A \cdot 2y \cdot \rho g$$
$$= -2Ay\rho g.$$

$$m = \text{volume} \times \text{density} = A(2h)\rho = 2Ah\rho.$$

acceleration $a = f/m$

$$a = -\frac{2Ay\rho g}{2Ah\rho} = -\frac{g}{h} \cdot y.$$

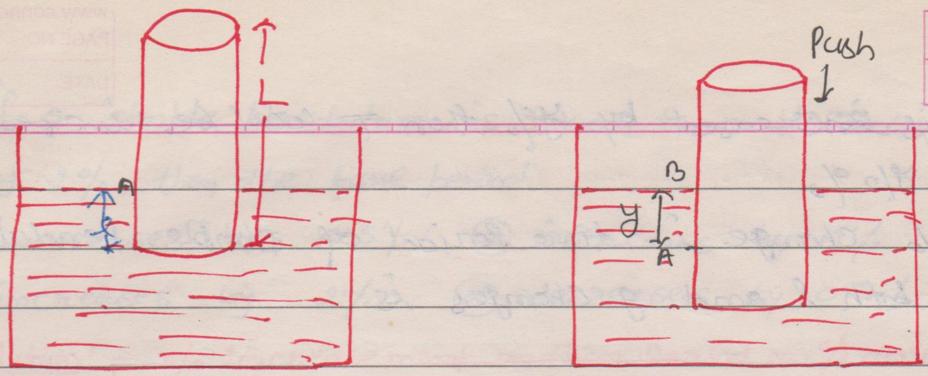
$$\boxed{T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{h/g}.}$$

* In Case of oscillations of a floating cylinder

$$\boxed{T = 2\pi\sqrt{\frac{L \cdot \rho}{\sigma g}} = 2\pi\sqrt{l/g}}$$

where L - vertical length of cylinder of density ρ and σ is the density of liquid in which cylinder is floating. l is the length of cylinder inside the liquid.

Solution! - Consider a cylindrical body of length L, cross-sectional area A floating in a liquid of density σ while floating, it up to point A, such that its length l is inside the liquid. Let ρ be the density of the material of the



floating body.

weight of the body $= AL\rho g$.

weight of the liquid displaced $= Al\sigma g$.

$\therefore AL\rho g = Al\sigma g$

$\Rightarrow L = l\sigma/\rho$.

$f = -Ay\sigma g$.

$$a = f/m = -\frac{Ay\sigma g}{AL\rho} = -\frac{\sigma g}{\rho L}\cdot y$$

$$\boxed{T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{\rho L}{\sigma g}} = 2\pi\sqrt{l/g}}$$

* If T_1 & T_2 are the time periods of a body oscillating under the restoring forces F_1 & F_2 then the time period of the body under the influency of resultant $\vec{F} = \vec{F_1} + \vec{F_2}$ will be

$$\boxed{T^2 = \frac{T_1^2 T_2^2}{T_1^2 + T_2^2}}$$

* The Percentage change in time period of simple Pendulum when its length changes is

$$\boxed{\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100\,\%}$$

* If l is increased by $x\%$ then T is increased by $x/2\%$.
* The percentage change in time period of simple Pendulum when 'g' changes but l remains Constant is

$$\boxed{\frac{\Delta T}{T} \times 100\% = \frac{1}{2}\frac{\Delta g}{g} \times 100\,\%}$$

* If g is decreased by y% then T will be increased by y/2 %

* The % change in time period of simple pendulum when both l and g changes is

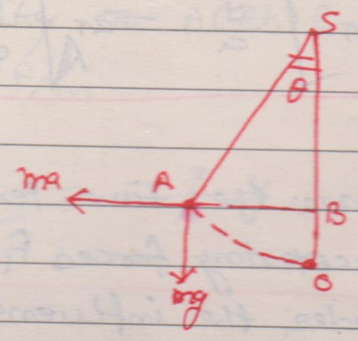
$$\boxed{\frac{\Delta T}{T}\times 100 = \frac{1}{2}\left(\frac{\Delta l}{l} + \frac{\Delta g}{g}\right)\times 100\%}$$

* A simple pendulum is set up in a trolley which moves to the right with an acceleration a horizontal plane. Then the thread of the pendulum in the mean position makes an angle with the vertical

$$\boxed{\theta = \tan^{-1} a/g \text{ in the backward direction}}$$

$\tan\theta = \frac{AB}{SB} = \frac{ma}{mg}$

$\boxed{\theta = \tan^{-1}(a/g)}$



* If a body is released into a tunnel dug along the diameter of the earth of radius R, it executes the S.H.M. with time period

$$\boxed{T = 2\pi\sqrt{R/g}}$$

* A body has a time period t_1 under the action of one force and t_2 under the action of another force, when both the forces are acting in the same direction the time period is t then

$$\boxed{t^2 = t_1^2 t_2^2 / t_1^2 + t_2^2}$$